GRID-SCALE AND SUBGRID-SCALE EDDIES IN TURBULENT FLOWS ~ A PRIORI TEST

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Abstract Grid-scale (GS) and subgrid-scale (SGS) eddies in homogeneous isotropic turbulence are identified by a priori test. GS and SGS velocity fields are obtained by filtering the DNS velocity field for different Re_λ using three classical filters for LES: Gaussian filter, tophat filter and sharp cutoff filter, and the most important filter width is considered as the length of Kolmogorov microscale in the DNS field with a constant multiplication. Eddies in the GS field, i.e., eddies larger than filter width are considered as grid-scale eddies and others are subgrid-scale eddies. Second invariant Q of the velocity gradient tensor is used for identification of these structures in GS and SGS fields. By visualizing the contour surfaces of second invariant Q, it is shown that GS and SGS fields itself contain lots of distinct tube-like eddies in homogeneous isotropic turbulence, which indicates that the DNS field may contain multi-scale structures in turbulent flow.

Keywords: Computational Fluid Dynamics, Direct Numerical Simulation, Large-eddy Simulation, Homogeneous Isotropic Turbulence

INTRODUCTION

Direct numerical and large-eddy simulations (DNS and LES) have been widely used to study the physics of turbulence. However, direct numerical simulation for high Reynolds number flow requires formidable computing power, and is only possible for low Reynolds numbers. Generally, industrial, natural or experimental configurations involve Reynolds numbers that are far too large to allow direct simulation, and the only possible method is large-eddy simulation.

It has been shown in recent years that turbulent flows contain various types of vortical structures and there is a large range of scales. During the past decades, the studies on the small-scale and large-scale structures in turbulence have been the subjects of considerable interest among turbulence researchers. The study on these structures is promising not only for understanding turbulence phenomena but also for modeling of turbulence.

In the theoretical study, it is believed that tube-like structure is a type of eddy or vortex, which is the candidate of fine scale structure, particularly, in the small-scale motions in turbulence (Tennekes, 1968; Lundgren, 1982; Pullin and Saffman, 1993). Nowadays, from direct numerical simulations of turbulence (She et al., 1990, Vincent and Meniguzzi, 1991, Jimenez et al., 1993; Tanahashi et al., 1997a, 1999a, Uddin et al., 2001), fine scale tube-like eddies in homogeneous turbulence are observed, and the visualization of this small-scale eddies in the turbulent flow becomes possible. In the recent studies, by direct use of local flow pattern (Tanahashi et al., 1997a, 1999a), the cross-sections of tube-like coherent fine scale eddies are investigated from DNS database of homogeneous isotropic turbulence in which the cross-sections are selected to include the local maximum of second invariant of the velocity gradient tensor on the axis of the fine scale eddies and then characteristics of the coherent fine scale eddies are identified. By applying the same analyses to turbulent mixing layer (Tanahashi et al., 1997b), it is shown that the characteristics of tube-like eddies in homogeneous isotropic turbulence and fully-developed turbulent mixing layer obey the same scaling law. In their study, the educed fine scale eddies have similar mean azimuthal velocity profiles and distinct axes, that is why, they described these eddies as ‘coherent fine scale structures’ in turbulence. The characteristics of vortical structures in turbulent channel flows (Tanahashi et al., 1999b) and MHD turbulence (Tanahashi et al., 1999c) also show the similar behavior of tube-like eddies in homogeneous isotropic turbulence. These results suggest that the existence of ‘coherent fine scale eddy’ in turbulence is universal.

Although DNS is the most exact approach to turbulence simulation but too expensive while Large-eddy simulation (LES) is less expensive and can

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simulate very complex flow fields in turbulence. With LES method, large-scale motion is directly calculated but small-scale needs to be modeled by subgrid-scale (SGS) model. Concerning the subgrid-scale model, it seems quite important to know what happened in the filtered field for LES including the appearance or disappearance of the tube-like vortical structures in resolved or unresolved field.

Therefore, the objective of this study is to filter DNS velocity fields for generating the grid-scale and subgrid-scale velocity fields in turbulent flow using classical filters for LES. Then we identified the grid-scale and subgrid-scale eddies to perform *a priori test* in homogeneous isotropic turbulence.

LARGE-EDDY SIMULATION

DNS Data Base

In this study, DNS data of decaying homogeneous isotropic turbulence have been used, which are conducted by Tanahashi et al. 1997a. Reynolds numbers based on u_{rms} and Taylor microscale, λ of the DNS data are Re_λ=64.9 and 87.9. The number of grid points for Re_λ=64.9 and 87.9 are 128^3 and 216^3, respectively.

GS and SGS Velocity Fields

To obtain the grid-scale (GS) and gubgrid-scale (SGS) velocity fields, we have directly filtered the DNS velocity fields using three classical filters: Gaussian filter, tophat filter and sharp cutoff filter for LES. In LES, a velocity component u can be decomposed into two components, one component is in the range of low wave-number of energy spectrum, called GS component and is denoted by $\overline{u}$, and the other component is in the range of high wave-number of energy spectrum, called SGS component and is denoted by u'. Their relation can be expressed as:

$$u = \overline{u} + u' \tag{1}$$

The filtering is represented mathematically in physical space as a convolution product (Leonard, 1974). The filtered part $\overline{u}$ of the variable u is defined formally by the relation:

$$\overline{u}(x) = \int_D G(x - x', \Delta)u(x')dx', \tag{2}$$

in which D is the entire domain, G is filter kernel function and Δ is the filter width. The dual definition in the Fourier space can be obtained by multiplying the spectrum $\hat{u}(\mathbf{k})$ of u(x) by the spectrum $\hat{G}(\mathbf{k})$ of the kernel G(x) such that,

$$\hat{\overline{u}}(\mathbf{k}) = \hat{G}(\mathbf{k})\hat{u}(\mathbf{k}), \mathbf{k} = 0, \pm 1, \pm 2, ... \tag{3}$$

The function $\hat{G}$ is the transfer function associated with the kernel G.

Classical Filters for LES: In this study, three classical filters are used for performing the spatial scale separation. The DNS fields are filtered in Fourier space (FS) rather than physical space (PS). For a filter width Δ_i in i-direction, these filters in FS are written as follows:

(I) Gaussian filter:

$$\hat{G}(k_i) = \exp\left\{ -\frac{(\Delta_i k_i)^2}{24} \right\} \tag{4}$$

(II) Tophat filter:

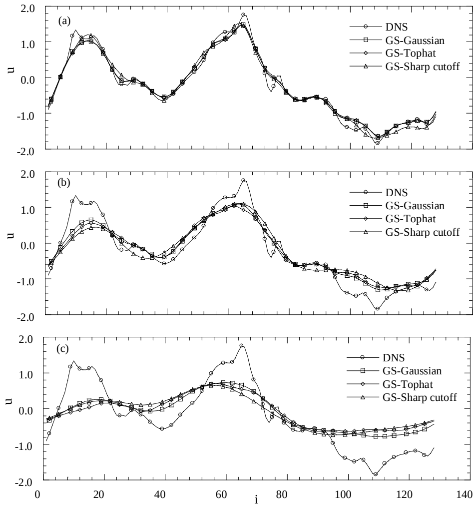

Fig. 1 Sample of filtered and unfiltered velocity fields for *Re*$_\lambda$=64.9. (a) $\overline{\Delta}$ =10η, (b) $\overline{\Delta}$ =20η, (c) $\overline{\Delta}$ =40η.

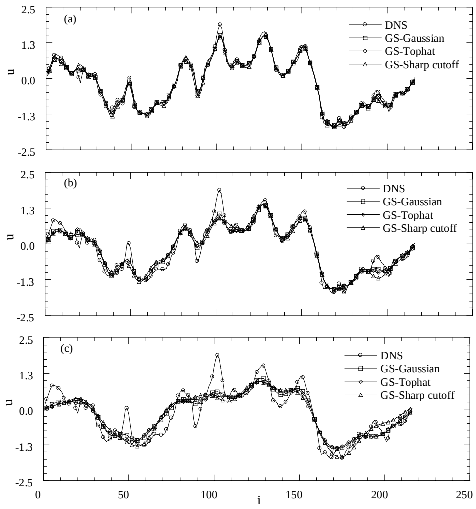

Fig. 2 Sample of filtered and unfiltered velocity fields for *Re*$_\lambda$=87.9. (a) $\overline{\Delta}$ =10η, (b) $\overline{\Delta}$ =20η, (c) $\overline{\Delta}$ =40η.

$$\hat{G}(k_i) = \frac{\sin\left(\frac{\Delta_i k_i}{2}\right)}{\left(\frac{\Delta_i k_i}{2}\right)} \tag{5}$$

(III) Sharp cutoff filter:

$$\hat{G}(k_i) = \begin{cases} 1, \left(|k_i| \le \dfrac{\pi}{\Delta_i} \right) \\ 0, \left(|k_i| > \dfrac{\pi}{\Delta_i} \right) \end{cases} \tag{6}$$

Using the above three filters for LES in the Fourier space, two sets of DNS data are filtered and the exact GS velocity fields, $\overline{u}$ are obtained. After generating $\overline{u}$, the SGS velocity field can be obtained by the relation:

$$u' = u - \overline{u} \quad (7)$$

Filter width plays very important role with filter functions in this process. The characteristic filter width

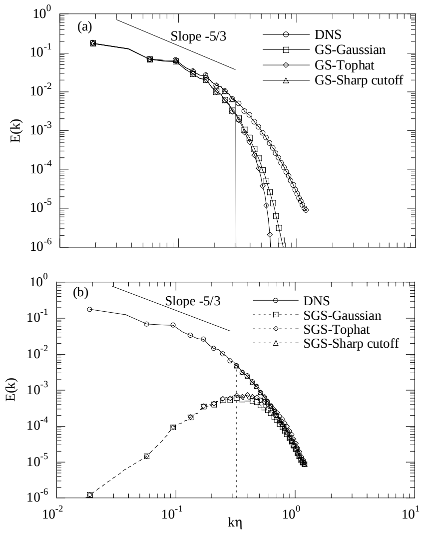

Fig. 3 Three-dimensional energy spectra of velocity fluctuations for Re_λ=64.9. (a) DNS and GS fields, (b) DNS and SGS fields. Filter width, $\overline{\Delta}$ =10η.

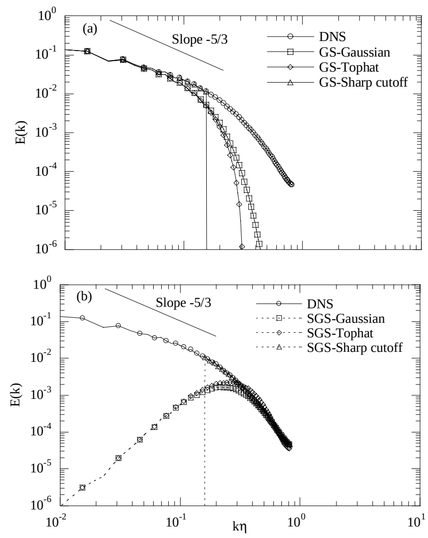

Fig. 4 Three-dimensional energy spectra of velocity fluctuations for Re_λ=87.9. (a) DNS and GS fields, (b) DNS and SGS fields. Filter width, $\overline{\Delta}$ =20η.

Δ_i is commonly used as the length, approximately proportional to the grid interval Δ in the previous researches (Piomelli et al., 1993, Horiuti, 1992). The structures represented by the GS and SGS velocities consequently depend both on the grid interval and on the type of filter employed. In the previous studies (Tanahashi et al., 1997a, 1999a), it is shown that the mean diameter of the coherent fine scale eddy is about 10 times of Kolmogorov microscale (η) in turbulent flows. Therefore, in this study, the most important filter width, Δ_i, is considered as the length of Kolmogorov microscale in the DNS field with constant multiplications. Since we are dealing with homogeneous isotropic turbulence, the filter width Δ_i is same in each direction and hereafter it is denoted by $\overline{\Delta}$.

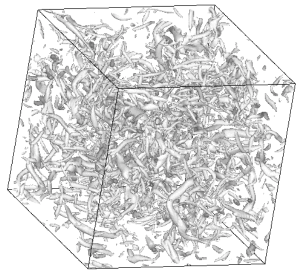

Fig.5 Contour surfaces of the second invariant of the velocity gradient tensor (Q*=0.03) for the case Re_λ=64.9 in the DNS field. Visualized region is the whole calculation domain.

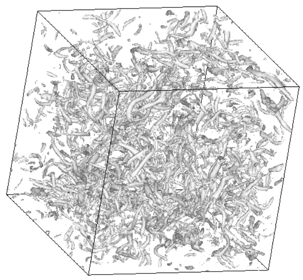

Fig.6 Contour surfaces of the second invariant of the velocity gradient tensor (Q*=0.03) for the case Re_λ=87.9 in the DNS field. Visualized region is 1/8 of the whole calculation domain.

Profiles of the DNS, GS and SGS Velocity Fields:

In order to understand the GS velocity field from the DNS velocity field, we have compared DNS velocity with filtered velocity using several filter widths. For this purpose we have randomly chosen one-dimensional velocity profile in x_1 direction and have plotted $u_1(x_1)$ and $\overline{u}_1(x_1)$ for Re_λ =64.9 in Fig.1 and for Re_λ=87.9 in Fig.2 using three filter widths, $\overline{\Delta}$ = 10η, 20η and 40η, where η is obtained from respective DNS velocity field. In all cases we obtained these one-dimensional profiles for the above three filter functions: Gaussian, tophat and sharp cutoff filter. The profiles in Fig.1 (a) suggest that the separation of SGS velocity field from GS velocity field for low Re_λ case is possible for filter width $\overline{\Delta}$ = 10η using all three filter functions, while $\overline{\Delta} \geq 40\eta$ do not represent the real GS velocity field. On the other hand it revealed from Fig.2 that the generation of GS velocity field is possible for high Re_λ case for filter width $\overline{\Delta} \geq 20\eta$, and $\overline{\Delta} \leq 10\eta$ shows almost similar profile as DNS (Fig.2 (a)). Fig. 1 and 2 clearly indicate that to separate SGS field from GS field in LES by filtering operation, filter width $\overline{\Delta}$ depends on the Reynolds number of the flow, regardless of the filter employed. For identification of GS and SGS eddies, in this study, we have considered $\overline{\Delta}=10\eta$ for Re_λ=64.9 and 20η for Re_λ=87.9.

Three-dimensional energy spectra of filtered and unfiltered velocity fluctuations for Re_λ=64.9 with $\overline{\Delta}=10\eta$ and for Re_λ=87.9 with $\overline{\Delta}=20\eta$ are presented in

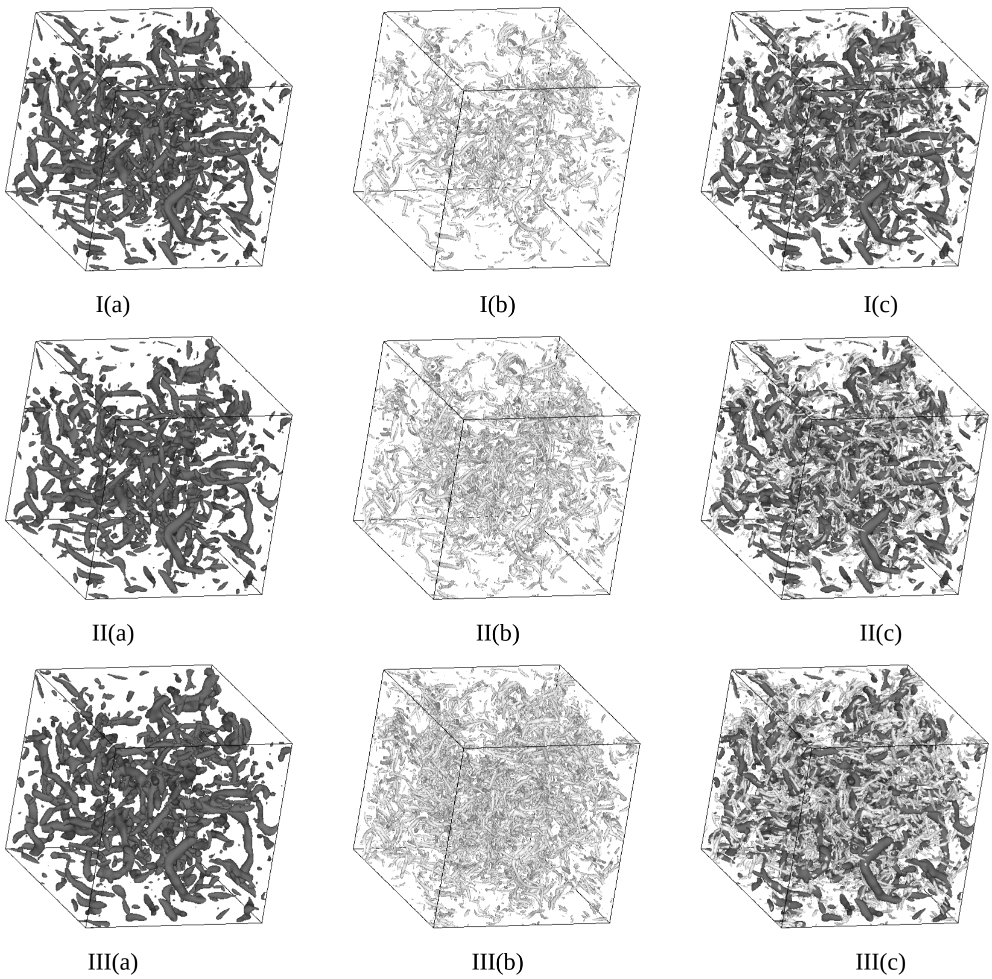

Fig. 7 Contour surfaces of the second invariant of the velocity gradient tensor in GS and SGS fields obtained from DNS field using (I) Gaussian filter, (II) tophat filter and (III) sharp cutoff filter for the case Re_λ=64.9. Visualized region is the whole calculation domain. (a) GS field (*Q=0.02), (b) SGS field (*Q**=0.01) (c) GS (*Q**=0.02) & SGS (*Q**=0.01) fields.**

Fig. 3 and 4, respectively, which is calculated by the definition written as follows:

$$E(k)=\sum_{k-\frac{1}{2}<|k|\le k+\frac{1}{2}} \frac{1}{2}\hat{u}(\mathbf{k})\hat{u}^*(\mathbf{k}) \tag{8}$$

In each case, the GS and SGS spectra are obtained by using Gaussian, tophat and sharp cutoff filter, and then compared with the DNS spectrum. The DNS spectra of two Re_λ cases show the power decay close to $k^{-5/3}$. In the previous study, using this DNS data (Tanahashi et al., 1997a), it is shown that the energy dissipation rate is dominated by the fine scale eddies in homogeneous isotropic turbulence, which is beyond the discussion of this paper.

With the sharp cutoff filter, the SGS fields contain the velocity due to all the structures with wave number $|k|>\pi/\overline{\Delta}$. On the other hand, full range of wave numbers contributes to the SGS velocity in case of the Gaussian and tophat filters. Figs. 3 and 4 reveal this difference in behavior for GS and SGS spectra using different filters for LES. These two figures clearly indicate that, when sharp cutoff filter is used, the GS spectra exactly collapsed with DNS spectra in the range of low wave numbers ($|k|\le\pi/\overline{\Delta}$), while SGS spectra in the high wave number range, and the contribution of subgrid scale is entirely due to the high wave numbers. The behavior of these spectra also confirms the accuracy of the filtering process. When Gaussian or tophat filter is used, the subgrid scales account for a fraction of the total (DNS) kinetic energy. Moreover, SGS energy has a significant contribution from the low wave numbers (i.e., the large scales) as we can see in

Figs. 3(b) and 4(b). However, with this small $\overline{\Delta}$, it seems that GS fields contribute to the large part of energy dissipation rate for all filter functions.

GS AND SGS EDDIES IN HOMOGENEOUS ISOTROPIC TURBULENCE

The concept usually associated with an eddy is that of a region in the flow where the fluid elements are rotating around a 'set of points'. Identification of the eddy or vortex from DNS/LES database is a very difficult and complex task, requiring considerable computational efforts with proper identification method.

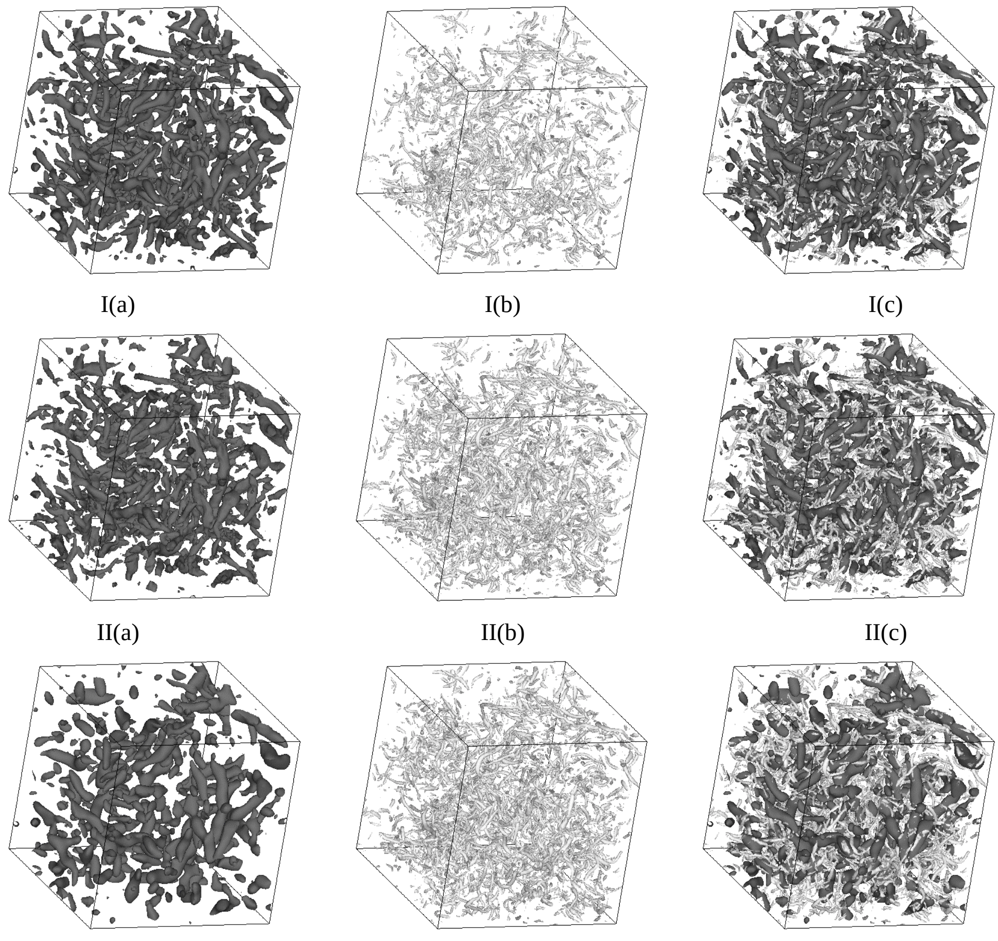

Fig. 8 Contour surfaces of the second invariant of the velocity gradient tensor in GS and SGS fields obtained from DNS field using (I) Gaussian filter, (II) tophat filter and (III) sharp cutoff filter for the case Re_λ=87.9. Visualized region is 1/8 of the whole calculation domain. (a) GS field (Q^*=0.005), (b) SGS field (Q^*=0.025) (c) GS (Q^*=0.005) & SGS (Q^*=0.025) fields.

There are several methods for identification of the vortical structures in turbulence with significant differences (Jeong and Hussain, 1995) and most of them show *threshold* dependence. It is necessary to consider an identification method that can educe the vortex or eddy without choosing any thresholds of variables. As we discussed in the introduction, direct 'local flow pattern' can educe coherent structures in several flow fields (Tanahashi et al., 1997a, 1999a, etc), which shows universal characteristics in turbulence. In our previous study (Uddin et al., 2001), using the same method we have identified the coherent fine scale eddies and its' axes without using any thresholds and then discussed the spatial distribution of coherent fine scale eddies by visualization of axes in homogeneous isotropic turbulence. Local flow pattern can best be determined by evaluating the second and third invariant of the characteristic equations of velocity gradient tensor in turbulence (Chong et al., 1990, Tanahashi et al., 1997a). The second invariant is defined as follows:

$$Q = -\frac{1}{2}\left(S_{ij}S_{ij} - W_{ij}W_{ij}\right), \qquad (9)$$

where S_{ij} and W_{ij} are the symmetric and antisymmetric part of the velocity gradient tensor A_{ij}:

In order to discuss about GS and SGS eddies, we notice the tube-like coherent fine scale eddy by visualization of flows in the DNS fields. Figs.5 & 6 show the contour surfaces of normalized second invariant of the velocity gradient tensor Q for Re_λ =64.9 and 87.9, respectively. The visualized region in Fig.5 is whole calculation domain while in Fig.6 it is 1/8 of the whole calculation domain. The level of the isosurface is selected to be Q^*=0.03. Hereafter, * denotes the normalization by Kolmogorov microscale η and root mean square of velocity fluctuations, u_{rms}. The normalization of Q by η and u_{rms} is due to the fact that the diameter and the maximum azimuthal velocity of tube-like fine scale structures can be scaled by η and u_{rms} (Tanahashi et al., 1997a). Figs.5 & 6 show that lot of coherent tube-like eddies are randomly oriented in homogeneous isotropic turbulence. However, if we increase or decrease the value of Q^*, we can also show distinct tube-like eddies in turbulence, little bit different from Fig.5 or 6, which means the visualization of fine scale eddies significantly depends on the value of threshold of Q (Uddin et al., 2001).

In this study, we discussed about GS and SGS tube-like eddies by visualization of flows in the GS and SGS fields, for different filter functions in homogeneous isotropic turbulence. Figs.7 & 8 show the contour surfaces of second invariant of the velocity gradient tensor Q in GS and SGS fields obtained by using Gaussian, tophat and sharp cutoff filters for Re_λ = 64.9 and 87.9, respectively. The visualized region and viewpoint in Fig.7 & 8 are same as in Fig. 5 & 6, respectively. The level of the isosurface in Fig 7 for all cases is selected to be Q^*=0.02 for GS fields and Q^*=0.01 for SGS fields. On the other hand, the level of the isosurface in Fig 8 is selected to be Q^*=0.005 for GS field and Q^*=0.025 for SGS fields. As we discussed above, the visualization of coherent structures depends on the threshold value of Q and we do not concern the strength of the structures, therefore, in this visualization we considered these different values for Q for different fields only to show the vortical structures in GS and SGS fields by visualization. Figs 7 & 8 clearly indicate that GS and SGS fields contain lots of distinct tube-like structures somewhat similar to DNS fields, which can best be defined as coherent eddies in turbulence (Uddin et al, 2001). Whatever the Re_λ or $\overline{\Delta}$, it is also clear that the size or length of GS eddies in all cases seems to be larger than SGS eddies.

It is known from the classical idea of fluid dynamics that several small-scale structure together form a large-scale structure, i.e., several small (SGS) eddies completely lie inside a large (GS) eddy in order to its size or strength. However, Fig. 7(c) & 8(c) clearly indicate that GS (dark) and SGS (light) eddies are quite distinct and unique in turbulence. In our previous study (Uddin et al, 2001), we have shown that the tube-like coherent fine scale eddies itself contain its distinct axis as well as local maximum of second invariant on the axis. The present study also suggests that the turbulence fields contain tube-like eddy in different size and length and each eddy has its distinct axis. This result will be worthy to develop a structure base SGS model for LES.

CONCLUSIONS

In this study, DNS velocity fields for different Re_λ are successfully filtered by using three classical filters for LES and then GS and SGS fields are obtained in homogeneous isotropic turbulence. By visualizing the contour surfaces of second invariant Q in GS and SGS fields as well as in DNS field, it is shown that GS and SGS fields itself contain lots of distinct tube-like eddies in homogeneous isotropic turbulence, which indicates that the DNS field contain multi-scale structures in turbulent flow. The study on detailed characteristics of these GS and SGS eddies are currently underway.

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